From linearity to nonlinearity and a maximum point, not to infinity

By V. Laxmanan, Sc. D

The following is a response to the interesting question posed by Prof. Derek Abbott.

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The paradox of infinity: where does the nonlinearity come from?

Ah, the wise Prof. Abbott's infinity paradox!

I do not understand infinity (only know the definition in the calculus course from college), or the paradox, that you perceive, but I do understand nonlinearity to some extent and here's my take since it was a) the subject of my doctoral thesis and b) I am discovering nonlinearity once again with experimental data, which, interestingly are all expressed nicely as integers. More about this shortly. Apologies, in advance, also for a lengthy post.

First of all, let me say that the less we understand a problem, the more seems to be our obsession with nonlinearity and the dismissal of any explanations based on linearity as being just too simple-minded. Just ask the politicians and economists and those screaming Cramers on Wall Street. They are obsessed, me thinks, with nonlinearity in their attempt to understand what are, of course, very complex and bewildering problems. But deep insights can be gained if we learn to simplify complex problems and study their individual components. Then, linear laws magically appear!

Indeed, the most profound laws of nature are linear laws, with few notable exceptions, like Kepler's third law (a three-halves power law, $y = kx^n$ where n = 1 for linear law and n = 3/2 = 1.5 for Kepler's law) and the laws that governed topic of my doctoral thesis. Here's a short list of linear laws: Ohm's law for voltage-current relation, Hooke's law relating stress and strain, Charles' law for an ideal gas, Einstein's photoelectric law, the hydrostatic law, relating pressure p to the depth below the free surface of a liquid (applies to all liquids). And there are many others.

In the study of fluids, especially more complex polymeric liquids (with complex chain like molecules), or liquid-solid mixtures, we find that the linear law relating shearing stress to the shearing rate, known as Newton's law of viscosity, breaks down. The slope of the straight line graph is called the viscosity and it is constant when the liquid is a "simple" one, like common engine or motor oils, or even water. In other words, "simple" systems produce linearity, while "complex" systems are nonlinear.

Thus, corrections are needed to this "simple" constant viscosity behavior and now we have nonlinearity, which was the topic I was alluding to. My thesis was aimed at characterizing the fundamental deformation characteristics of a novel material, with a unique microstructure, in the limit of low shearing rates, where the viscosity becomes very large. This was the basis of an innovation in materials processing which has led to new manufacturing processes to produce a variety of metal alloy components. The graduate students who preceded me discovered this process and had studied the high shearing rate regime. Nonlinearity was evident in their studies and the material also exhibited a complex behavior called the hysteresis loop, which means that when x increases y increases but when x is decreased the y values do not follow the same straight line or curve and trace a complete loop that closes upon itself on a x-y plot.

This is also observed during magnetization and demagnetization of materials and the area of the loop (in what is called the B-H diagram) is a measure of the energy lost in this up and down process. An ideal material for a transformer must have a small B-H loop and this too was a topic that I studied in my first job after finishing my thesis.:)

At first, in my thesis, we thought that we might see linear behavior. But, nonlinearity was evident and my thesis advisor declared that I should derive the equations that describe the nonlinearity to analyze my experimental data more critically! Up your alley, he said! I was taken aback. Anyway, I did derive them which were checked by another senior graduate student of his (who was already in academia and had authored a popular textbook on this topic). And, so I have to say I understand nonlinearity and the paper that we published out of my thesis is still being widely cited.

And now, recently, I have again discovered nonlinearity and now we are dealing with (x, y) values which are all expressed neatly in whole numbers. (We did NOT have whole numbers in my thesis work.) So, this might just be the 'food for thought" that Prof. Abbott and all the folks here might find to be of interest.

Whether this leads to some kind of an infinity paradox, I don't know, but what I found was that nonlinearity leads to a maximum point. There is NO divergence to infinity!

That is also what happened with blackbody radiation. A maximum point appeared, as a result of nonlinearity, and had to be explained. Quantum physics was born. Physics is different from mathematics and some of the questions posed in this forum seem to capture this dilemma which has escaped/engaged even the greatest minds in science.

Here's what I found in the last few days. I plan to upload the full article shortly. I got interested in the topic of global warming recently. The bitter cold spell we are experiencing in the US got to me! Among other things that I started analyzing was the precipitation (P) data and its relation to the local average temperature (T). Here are some numbers for Russia obtained from http://www.russia.climatemps.com/

City No. 3 in the list (Cape Celjuskin/Fedo), T = 4 and P = 247

City No.14 (Dudinka), T = 14 and P = 529

Nice straight line between these (x, y) pairs and P = 28.2T + 134.2

There go all the whole numbers out the door!

Fractions have appeared already. Anyway, let us prepare a graph of the T and P values. We see an interesting behavior. The two cities here define what looks like the "maximum precipitation" line. For the same average temperature T, depending on the latitude, altitude and other geographical factors (in other words, complexity), we find that there is a maximum precipitation P. The two cities that I have listed define the limiting straight line. All the T and P data fall below this line. And, remember, we are dealing with whole numbers for both T and P, hence, my post, since the question starts with "whole numbers adding up", unless I have very much misunderstood things in which case, my sincere apologies for this post.

Nonetheless, it is interesting to note how the P-T diagram evolves as we start adding more and more data. Although there is a deviation from the straight line defined by the maximum P-T line, we see no evidence of nonlinearity at first. There is more scatter but the data can still be satisfactorily described using a linear law, or the best-fit line deduced using least squares method. However, when we get to 35 weather stations, there is a clear evidence for nonlinearity and what appears to be a maximum point. Continue this for all the cities and there is no mistaking the appearance of a maximum point. Of course, some of the new data added begins to approach and fall very close to the max P-T line identified above. But, nothing crosses that maximum P.

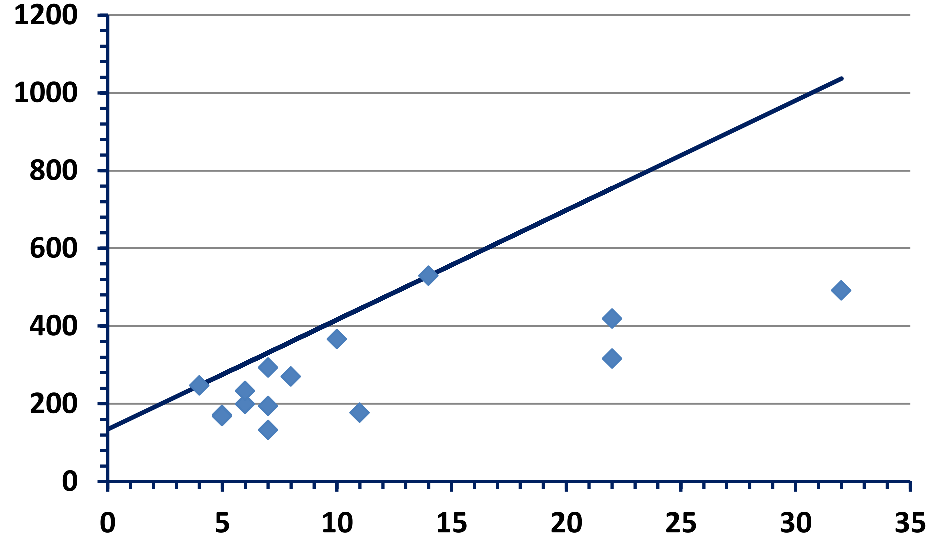

Figure 1: Graph of the local average temperature T (horizontal axis) versus the average annual precipitation P (vertical axis) for the first 15 weather stations for the Russian climate data, see http://www.russia.climatemps.com/ for the data; through Murmansk (68° 57'N). Cities in order decreasing latitudes.

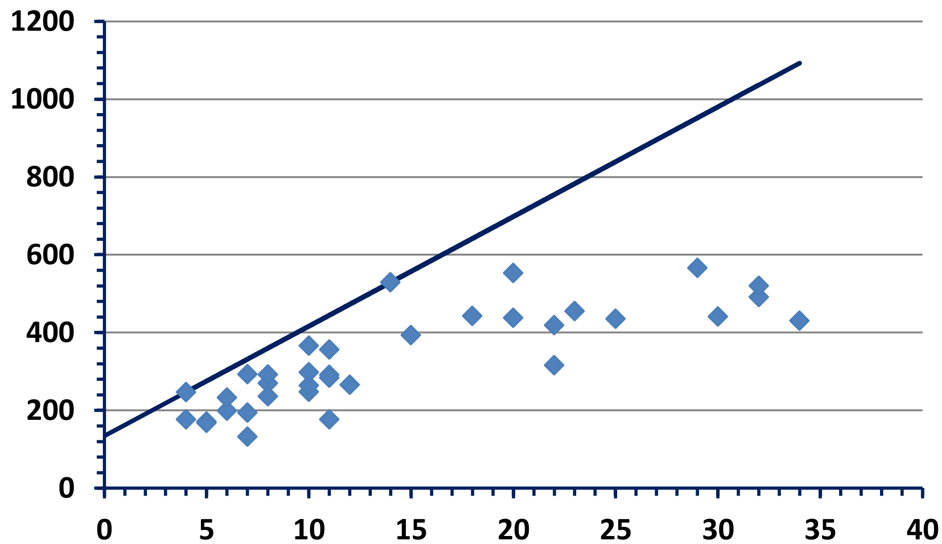

Figure 2: Graph of the local average temperature T (horizontal axis) versus the average annual precipitation P (vertical axis) for the first 35 weather stations for the Russian climate data, see http://www.russia.climatemps.com/ for the data; through Ust-Clima (65° 26'N). A maximum point is now evident.

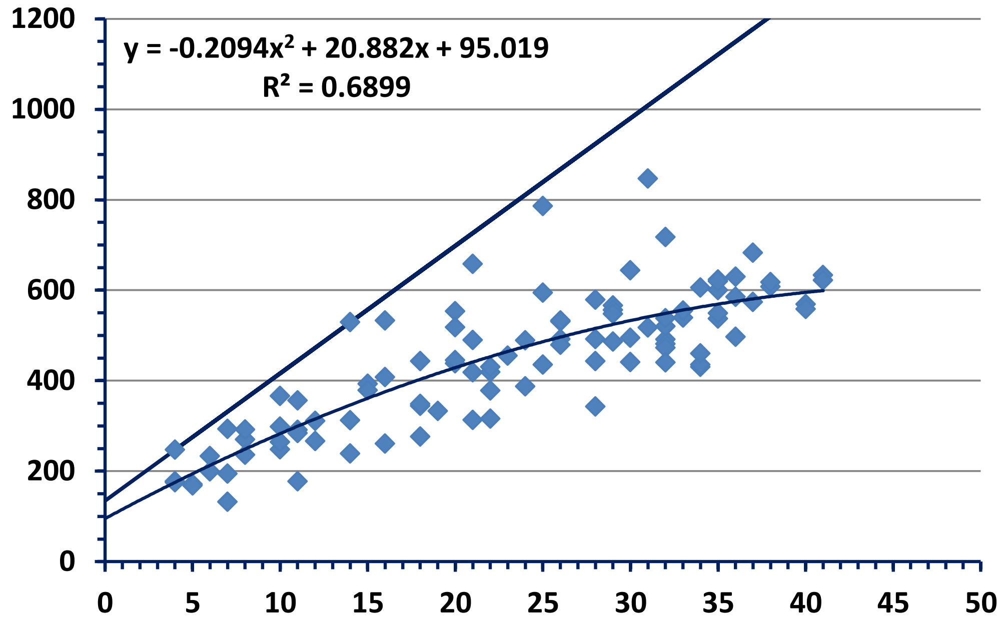

Figure 3: Graph of the local average temperature T (horizontal axis) versus the average annual precipitation P (vertical axis) for the first 100 weather stations for the Russian climate data, see http://www.russia.climatemps.com/ for the data; through Tomsk (56° 30'N). The nonlinear (polynomial) curve was generated by the Microsoft Excel program by clicking on "Add trendline" and then choosing the polynomial option. Some additional (x, y) pairs are now seen to fall close to the maximum P-T line identified in Figure 1.

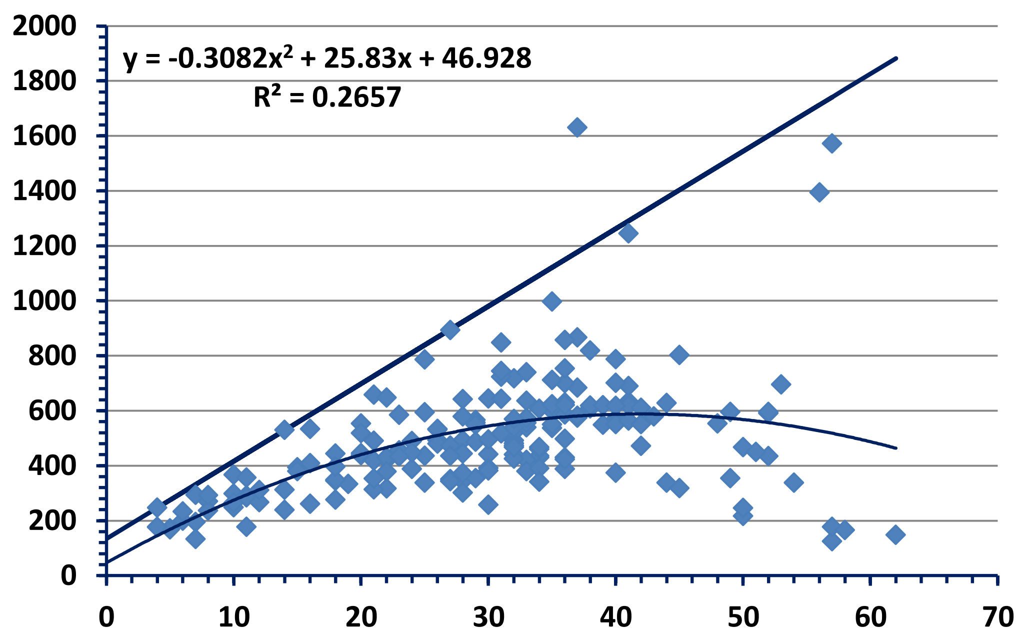

Figure 4: Graph of the local average temperature T (horizontal axis) versus the average annual precipitation P (vertical axis) for ALL 184 weather stations for the Russian climate

data, see http://www.russia.climatemps.com/. The nonlinear (polynomial) curve was generated by the Microsoft Excel program by clicking on "Add trendline" and then choosing the polynomial option. Some additional (x, y) pairs are now seen to fall close to the maximum P-T line identified in Figure 1. Also, we see evidence now for a maximum point, which is confirmed by the mathematical equation for the parabolic (polynomial) trendline.

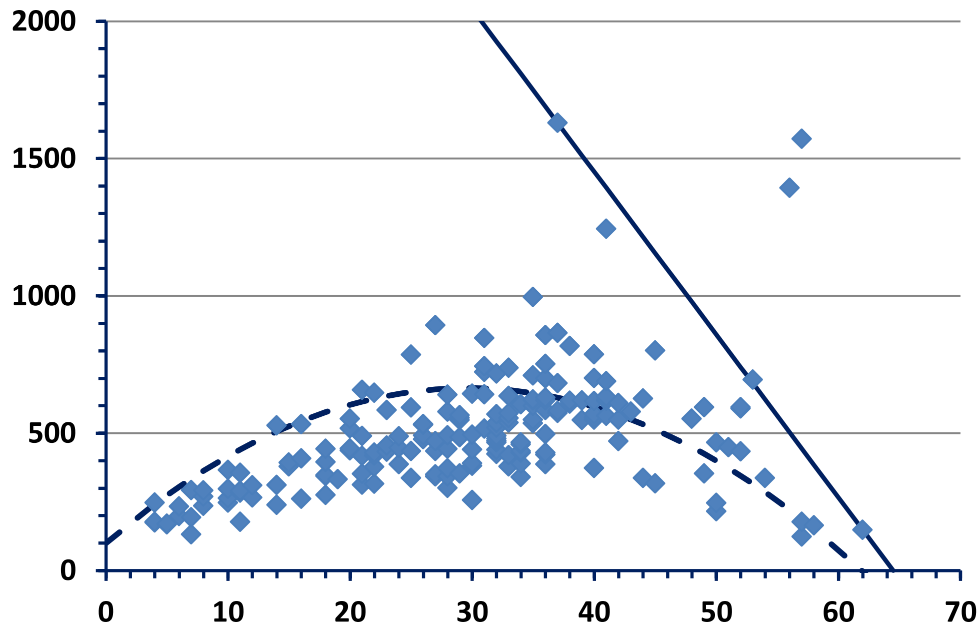

Figure 5: Re-analysis of the same data with a straight line, P = -59.28T + 3823.36, with a negative slope to define the maximum precipitation at any temperature T. Also, the parabola, $y = ax^2 + bx + c$, with the constants a = -0.64, b = 38 and c = 100, is include to highlight the existence of a maximum point of the graph. This parabola, unlike the parabola of Figure 4 is NOT based on any statistical arguments. It is simply an attempt to explain the observations.

This maximum precipitation P, at any fixed T, and the appearance of a maximum point on the P-T graph, as opposed to (a divergence to) the infinity paradox, IMHO, is of great fundamental significance for climate science, the data that I am analyzing now; hence also my desire to share these ideas with this august group of researchers, from many different disciplines.

The maximum P that I have been calling attention appears to me like the maximum electron energy that Einstein talks about in his photoelectric law. Only the determination of the MAXIMUM kinetic energy of an electron, produced by a photon, will yield accurate values for the Planck constant. If E = hf is the energy of a photon, the maximum energy of the electron is K = E – W = hf – W, where W is the energy that must be given up to bring the electron out of the metal and overcome the forces binding it to the metals. Millikan reviews these points in his first 1916 paper on photoelectricity, and as we know, he was able to deduce the value of the Planck constant h, from the slope of the (x, y) graph for his experiments. Indeed, Millikan avoids statistical methods entirely and simple takes several (x, y) pairs and finds the average slope to get the numerical values of a universal constant - which is tied to the integral values of an elementary unit of energy.

Paradoxical isn't it?

Starting with (x, y) values which are NOT whole numbers, Millikan finds a slope which is tied to the integral multiples of a fundamental unit of energy.

A generalization of the Planck-Einstein ideas to problems outside physics, has been my interest in recent years, and this also seems to apply to global warming science, based the P-T data analysis alluded to here. In the climate data, we start with whole numbers and find a slope that defines some kind of a maximum in this process. And, we also see a maximum point like they found in radiation physics. In fact, I have shown that the P-T data can be fitted to the same equation, $y = mx^n\exp(-ax)$, which is known as Wien's law. Instead of the slightly more complicated Planck law, Einstein decides to use the simpler law in his 1905 paper on what is now called the quantum theory of light. After showing radiation, in the form of light, behaves like discrete particles which have the elementary unit of energy conceived by Planck, he applies this to explain the photoelectricity puzzle and the cut-off frequency that was discovered by Lenard. Thus came the photoelectric law, the idea of a work function W in physics, and the determination of a universal constant h, tied to integral and whole units of energy.

A broad generalization of the Planck-Einstein ideas, to problems outside physics, is possible and has engaged my attention over the last few years, as also evident from the P-T data analysis being alluded to here. Perhaps, the methods of quantum physics can be applied to explain the P-T data, and many other such observations from many complex systems. I see not the infinity paradox, but the appearance of a maximum point; hence this post.

V. Laxmanan
February 9, 2014

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