ON THE GENERALIZATION OF EINSTEIN'S IDEA OF THE PHOTOELECTRIC WORK FUNCTION: EXAMPLE FROM SCANDINAVIAN CLIMATE DATA

By V. Laxmanan, Sc. D.

Abstract

A linear law, with a nonzero intercept c, of the type $y = hx + c = h(x - x_0)$, is often observed when we analyze our (x, y) observations on a number of complex systems. The climate system data is considered here for illustrative purposes. The nonzero c in such a law is like the nonzero work function W, conceived by Einstein, in 1905, to explain the photoelectric effect. Einstein's law was thus able to explain the cut-off frequency observed experimentally by Lenard. Likewise, there is a cut-off $x_0 = -c/h$. The photoelectric law implies a movement of the empirical observations along a family of parallel lines. A similar movement along parallels is observed if we analyze our (x, y) observations carefully. The method of deducing the existence of such parallels is also discussed and is traced to the method used by Millikan to determine the two universal constants: the absolute magnitude on the charge q on a single electron and the Planck constant h.

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Email address: vlaxmanan@hotmail.com . The author has recently created a Facebook group called Global Warming for the Layman; see https://www.facebook.com/groups/GWforlayman/, on January 5, 2014, aimed at discussing global warming data in an easy-to-understand manner, with short posts. He is a retired research professional, with advanced degrees in Mechanical Engineering and Materials Science and Engineering who has spent his entire professional career in leading US research institutions, in academia (MIT and CWRU), in government (NASA), and in corporate research labs (Allied Chemical Corporate R & D, now part of Honeywell, and the General Motors Research Labs). He has also published many widely cited scientific articles in leading peer-reviewed international journals in both physics and the materials sciences; see the few references cited at the end of this article.

Introduction

The main purpose here is to describe a nice example of a generalization of the idea of a work function, first conceived by Einstein in his famous 1905 paper on photoelectricity [1]. We will discuss an example from the climate sciences for this purpose. A brief explanation about photoelectricity and the meaning of the work function is provided in the next section followed by a discussion of the climate data (specifically, the average annual precipitation and the average local temperatures at various weather stations) for Norway and its Scandinavian neighbors.

Einstein's photoelectric law

In the 1905 paper [1-3], Einstein shows that a property of light radiation, called its entropy [3], permits us to think of a fixed volume of light as being made up of discrete particles, each having the elementary quantum of energy, conceived by Planck [4], just a few years ago, in 1900. So, when light shines on the surface of a metal, we can imagine a stream of photons striking the surface of the metal, each having the energy $E = hf$ where h is the Planck constant and f the frequency (of the light "wave"). Actually, in such an experiment, electrons with a wide range of energies are produced. However, Einstein says that the MAXIMUM kinetic energy of the electron $K = E - W$ where W is the energy that must be given up to bring the electron out of the metal, see Millikan [5-7] and overcome the complex forces that bind it to the metal.

Thus, $K = hf - W = h(f - f_0)$ and the K-f graph will be a series of parallels, each with the slope h, if we perform experiments with different metals; see the illustration in physics hypertext [8, 9]. In his Nobel Prize winning experiments [5], Millikan first determines the absolute magnitude of the electrical charge q on a single electron [10, 11]. This enables him to determine the MAXIMUM value of $K = V_0q$ of the electron and hence the Planck constant h from the photoelectric measurements with two metals, lithium and sodium. In these experiments, Millikan determines the stopping potential V_0 that leads to a zero photoelectric current. A collector cup surrounds the metal that is being irradiated with light of various frequencies. The ejected electrons reach this cup and flow in an external circuit to generate the photocurrent. By slowly making the collector more and more negative, the electrons can be repelled and the photocurrent ceases. Only the most energetic electrons will reach the collector. Thus, Millikan says that he has accurately determined the maximum K, exactly as emphasized by Einstein [2]. Only such a determination of the maximum K would give meaningful results for the Planck constant h. Indeed, in the introduction to the first 1916 paper on this subject, Millikan is careful to review all prior (not very successful) attempts to test Einstein photoelectric equation, $K = ½ mv^2 = V_0q = (hf – W)$. Here m is the mass of the electron and v its velocity which can be equated to the energy gained in moving the electron in the applied electric field.

However, interestingly, Millikan does NOT present the V_0-f graphs for lithium and sodium on a single V_0-f plot, not even in his Nobel lecture [5], to demonstrate the parallelism that is illustrated in [8].

Also, it is of interest to note that Millikan does NOT use statistical arguments [12-18] to determine either q or h. He simply finds the slopes between various (x, y) pairs on his graph for lithium and sodium and then determines an average slope to arrive at the value of h. Indeed, in the first 1916 paper Millikan arrives at the Planck constant h from just two (x, y) pairs for lithium.

Likewise, in the oil drop experiments, Millikan deduces the absolute value of q from the slope of the V-Q graphs for various drops. Here Q = Nq is the total charge on the drop, N the number of electrons attached to the drop, and V is the velocity with which the electrified oil drop is seen to move, either up or down, under the combined action of the earth's gravity field and the externally applied electric field. Millikan determines the velocity V accurately. Then, by considering various (x, y) pairs on the V-Q graphs for a number of drops, he shows that there is a single q that explains all his observations.

Thus, no statistical arguments, not even the least squares method [12], were used by Millikan to determine the two fundamental and most important constants of nature: q and h. The treatises by Longair [19] and Shamos [20] and the recent reviews of Planck's theory [21, 22] are highly recommended for further details. The cut-off frequency $f_0 = W/h$, when $K = 0$, first observed by Lenard [23], who received the Nobel Prize in 1905, the year Einstein's published his famous explanation, is actually a manifestation of the work function W. If $E < W$, no electrons will be observed since $K < 0$.

Planck's theory, which was the basis for Einstein's formulation of the discrete energy quanta to explain photoelectricity were preceded by Wien's work on the laws of heat radiation, also honored with the Nobel Prize [23, 24]. Indeed, Wien's simpler version of Planck's law, $y = mx^n e^{-ax} /[1 + be^{-ax}]$, was used by Einstein. For Wien's law $b = 0$ and for Planck's law $b = -1$. The equation given above is a generalization of both the Planck and Wien laws. The denominator was added by Planck since experimental observations seemed to yield better agreement with this modification of Wien's law, $y = mx^n e^{-ax}$. It could also be derived theoretically using statistical arguments. (Planck relates the notion of entropy of radiation to the elementary laws of combinations and permutations to determine the distribution of N particles among various energy states; see English translation of Planck's original 1900 paper in Shamos [4, 20].)

In what follows a case is being made for the generalization of the idea Einstein's work function to understand many complex problems outside physics, as also discussed in detail elsewhere [26, 27] with examples from financial, economic, social and political systems, sports, and the medical and environmental sciences. More recently, this generalization has also been discussed within the context of climate science, and the recent findings of a general stalling of global warming in the 21st century, which has been discussed in recent articles, both in the scientific literature and in the popular media. Here we will consider the example of precipitation and temperature data for Norway and its Scandinavian neighbors, Sweden, Finland, and Denmark to illustrate this generalization.

Norway Precipitation and Temperature data

The raw data being analyzed here has been compiled in Table 1 and was obtained from climatemps.com on February 9 and 10, 2014. The data was sorted by both increasing temperatures and precipitation values and examined carefully. If we limit ourselves to the weather stations with precipitation values P < 1200 mm, we see a nice linear trend, as illustrated in Figure 1. The solid line labeled I is the line joining the data for weather stations 2 and 15 and considers just the two (x, y) pairs to fix the slope a and the intercept b in the equation P = aT + b. This is exactly the method used by Millikan [6, 7]. The justification for choosing line I becomes even more obvious when we consider ALL of the data, as illustrated in Figure 2.

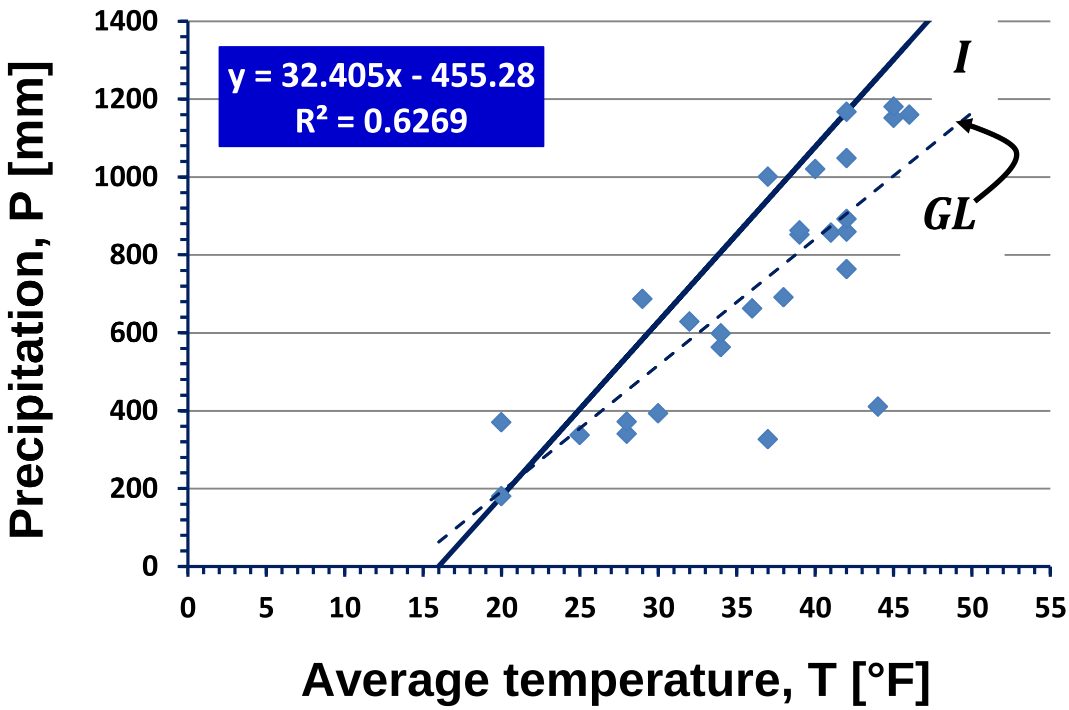

*Figure 1: The temperature-precipitation data for **28 cities of Norway** obtained on February 9, 2014 from http://www.norway.climatemps.com/ The solid line labeled I, with the equation P = aT + b = 44.864T – 717.273 = 44.86 (T – 15.99) joins the data for station 2, Svalbard Lufthavn (20, 180) and Brønnøysund, Nordland (42, 1147), station 15. It represents what appears to be the MAXIMUM precipitation observed at various temperatures, with very few exceptions; see also Figure 2. The dashed line is the statistical fit, the least squares line generated by the Microsoft Excel Program by clicking on "Add trendline" with "linear" option.*

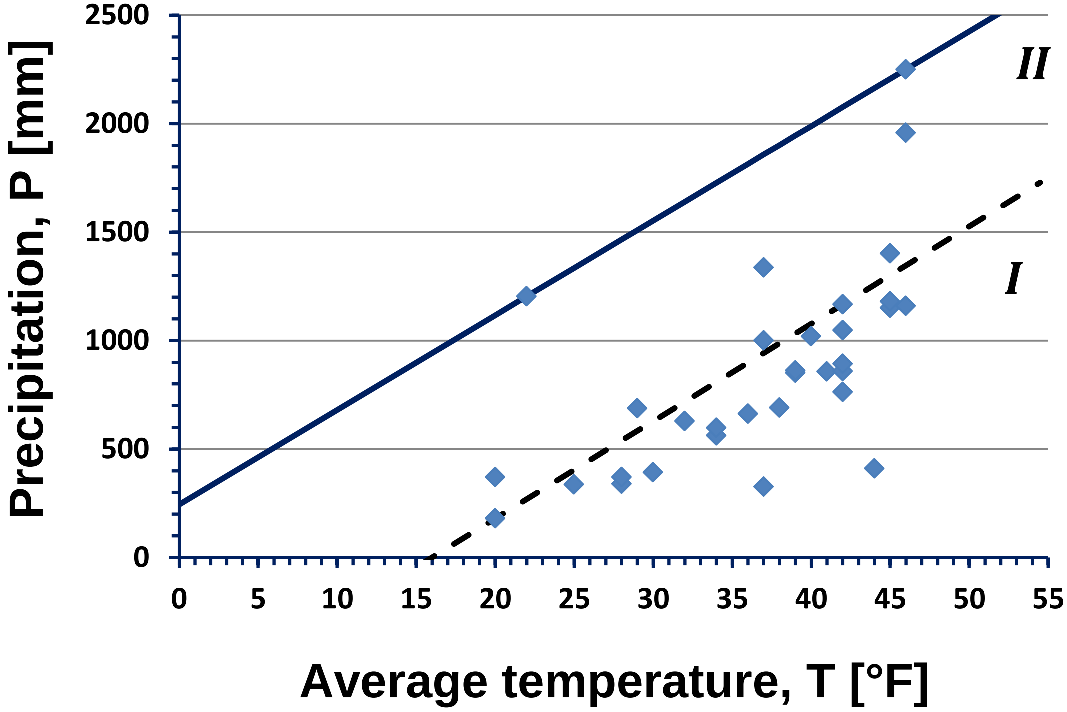

*Figure 2: The temperature-precipitation data for **ALL 32 cities of Norway** obtained on February 9, 2014 from http://www.norway.climatemps.com/ The dashed line labeled I, with the equation P = aT + b = 44.864T – 717.273 = 44.86 (T – 15.99) is the line of Figure 1. The solid line labeled II joins the data for station 22, Fannaråki/ Fanaråken, Sogn Og Fjordane (22, 1204) and Bergen - Florida (46, 2250), station 27. This line, P = 43.58T + 245.17 represents the MAXIMUM precipitation observed at each temperatures. Variations in the latitude, altitude and other geographical and climate factors are, no doubt, responsible for the range of precipitation values at each temperature. However, a simple linear law applies if we consider the maximum precipitation values. Statistical methods are thus avoided.*

The solid line labeled II in Figure 2, joins the two weather stations, see names in the figure caption, reporting the MAXIMUM precipitation. This line has virtually the same slope as the dashed line I of Figure 2, which is the same as that illustrated in Figure 1. In other words, physical arguments, rather than statistical arguments are being used to determine the constants “a” and “b” in what is clearly a simple linear law P = aT +b relating these two important climate variables.

Table 1: Precipitation and temperature data for Norway

City (with latitudes)	City No.	Average temp [°F]	Precipitation [mm]
Ny-Alesund (78° 55'N)	1	20	370
Svalbard Lufthavn	2	20	180
Isfjord Radio, Svalbard	3	25	337
Bear Island/ Bjørnøya, Svalbard	4	30	393
Bjornoya (74° 31'N)	5	28	371
Nordkap/Helnes Fyr (71° 4'N)	6	36	662
Jan Mayen Island (67° 16'N)	7	32	628
Jan Mayen B (70° 56'N)	8	29	687
Vardo, Finnmark (70° 22'N)	9	34	563
Tromso (69° 41'N)	10	37	1000
Karasjok (69° 28'N)	11	28	340
Narvik, Nordland (68° 28'N)	12	39	852
Bodo, Nordland (67° 16'N)	13	40	1020
Mo I Rana, Nordland (66° 21'N)	14	37	1337
Brønnøysund, Nordland (65° 28'N)	15	42	1167
Orland (63° 42' N)	16	42	1048
Vaernes (63° 28' N)	17	42	892
Trondheim (63° 25' N)	18	41	857
Kristiansund, Vestlandet (63° 7' N)	19	45	1151
Kråkenes, Sogn Og Fjordane (62° 2' N)	20	46	1160
Vagamo, Oppland (61° 52' N)	21	37	326
Fannaråki/ Fanaråken, Sogn Og Fjordane	22	22	1204
Lillehammer (61° 5' N)	23	38	691
Lærdal, Western Norway (61° 4' N)	24	44	410
Geilo (60° 32' N)	25	34	598
Bergen (60° 24' N)	26	46	1958
Bergen-Florida (60° 23' N)	27	46	2250
Gardermoen (60° 12' N)	28	39	862
Oslo (59° 57' N)	29	42	763
Dalen, Telemark (59° 27' N)	30	42	859
Stavanger, Rogaland (58° 53' N)	31	45	1180
Kristiansand (58° 10' N)	32	45	1401
Norway average		**37**	**860**

Data source: http://www.norway.climatemps.com/ obtained on Feb 9, 2014

Discussion

The focus on the maximum precipitation here is akin to the focus on the maximum energies of the electron in the photoelectricity experiments of Millikan. A range of electron energies is also observed in the latter experiments and inconsistent results for the Planck constant h would, obviously, be obtained if we fail to recognize the importance of the maximum kinetic energy of the electron. Likewise, a simple law relating the precipitation and the local average temperature can be deduced, by avoiding statistical arguments, and focusing on the maximum precipitation values after preparing the simple P-T diagram as illustrated here.

The nonzero intercept "b" in the law $P = aT + b = a(T - T_0)$ where $T_0 = -b/a$ is akin to the work function W in Einstein's photoelectric law. The temperature $T_0 = -b/a$ at which the precipitation will go to zero is akin to the Lenard cut-off frequency that could be explained by Einstein' idea of a particle nature for light combined with far reaching notion of a nonzero work function. While attention has been focused on the constancy of the slope h, and direct determination of the universal Planck constant from the photoelectric measurements, the equally important and far reaching notion of the work function W in Einstein's law has escaped attention to date. The Lenard cut-off frequency $f_0 = W/h$ is a manifestation of the work function W. Likewise, there is a work function, the name that can be given to the nonzero intercept in the linear law, that is lurking behind many other complex problems of interest to us. The climate science data, more completely discussed in recent articles [28-41], is just one such example. The Norwegian precipitation-temperature data considered here illustrates this point. Similar observations, perhaps, a bit difficult to comprehend since movement along parallel lines seems such a ridiculously simple, or rather a very simple-minded idea compared to sophisticated stochastic and statistical arguments that are generally used to analyze such data. However, as the well-known adage, "Lies, damned lies, and statistics" tells us, we only like our own statistics (or numbers) and condemn all else as simply lies.

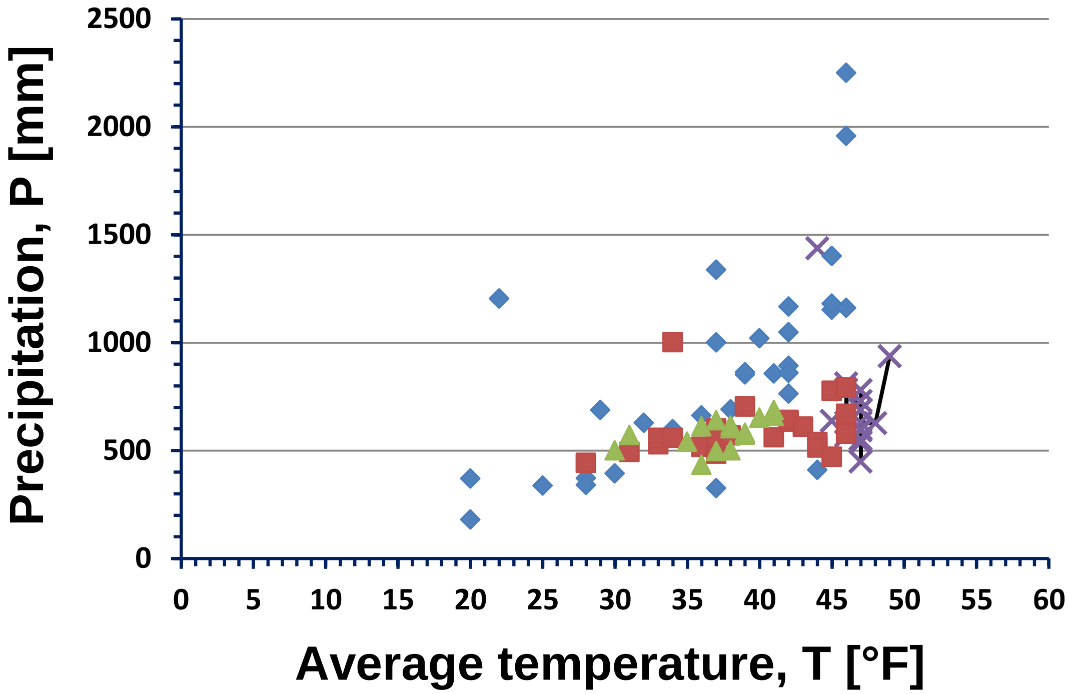

Figure 3: The composite plot of the precipitation data for Norway (◆) Sweden (■), Finland (▲) and Denmark (X). Again, the maximum precipitation for a fixed temperature is defined by the Norwegian stations, with the data for all stations from the other countries merely adding to the "scatter" in the plot. Also, interestingly, if we consider the data for low precipitation levels ($P < 1000$ mm, or $P < 1500$ mm), we see evidence for a maximum point on the graph. The data can thus be fitted to the power-exponential law $y = mx^n e^{-ax}$ with a maximum point at $x = x_m = n/a$, since $dy/dx = (n - ax)(y/x)$. This would represent an example of the application of Wien's equation, outside radiation physics.

The near complete dominance of statistical methods, outside the realm of physics, to the detriment of other analytical methods, is somewhat unfortunate and only leads to confusion and divisive debates, as we see with the ongoing climate science debate, and the famous Reinhart-Rogoff debate (on the debt to GDP ratio for various countries) that engaged the attention of many leading economists in 2013 and spilled into the popular media. These points, including a discussion of the debt/GDP ratio, is presented in [26,27].

For completeness, all of the (P, T) data for the Scandinavian countries, Sweden, Finland, and Denmark are plotted together with the Norwegian data in Figure 3. The P-T data for these three countries, if considered separately, seem to be confusing at best (see Figures 4 to 6). However, the composite plot of Figure 3, together with the analysis of Figures 1 and 2 supports the idea of a simple linear relation P = aT + b to describe all of this data, if we pay attention to the MAXIMUM precipitation. It also provides clues to a fuller understanding of the precipitation-temperature relationship, since a maximum point is revealed if we consider the low precipitation data; see also the discussion in the Appendix which provides the justification for the appearance of such a maximum point.

It is hoped that the discussion here will prove to be useful not only to climate scientists but also to economists, financial scientists, and other professionals in many disciplines who are now engaged in the analysis large volumes of data using nothing more than purely statistical arguments.

Finally, the present analysis is NOT a polemic against statistical methods. Far from it. As is well-known, from a study of the history of statistics [13, 14], Legendre's method became the most widely accepted method within just a couple of decades after its publication in 1805. Gauss had also used the same least squares method, much earlier (although the work was not published by Gauss), to predict the future positions of the asteroid Ceres [15-18] after it went missing (behind the glare of the sun), following its momentous discovery on New Year's Day of 1801. However, the success of Gauss should be interpreted NOT as the triumph of the statistical methods alone but as the judicious application of both physics and statistics.

Kepler's laws of planetary orbits, see Longair's treatise [19], had already been formulated. Gauss was able to incorporate these physical laws into the statistical method that he introduced (and also used by Legendre, independently). Nonetheless, the pitfalls of brute-force, or blind, application of statistical methods must be recognized. Unfortunately, over the last two centuries, statistical methodology seems to have become the scientific methodology, especially in the medical sciences and social sciences that intersect with medical science, such as the use of drugs, and study of problems such as teen age pregnancy, obesity, etc. This can be seen by examining any issue of prestigious medical journals such as JAMA (Journal of the American Medical Association).

What has been called the Millikan method here, and the physical arguments that must be invoked to understand complex problems, has largely been overlooked. The determination of the two universal constants q and h, without the aid of any statistical methods, not even the simple linear regression analysis (or least squares method, or the "best-fit" line) is noteworthy. This is the motivation for the analysis of the P-T data presented here, and becomes even more compelling when we consider the movement of global average temperature data, versus time, along a similar family of parallels. The author has identified a family of five such parallels, by a re-analysis of the temperature anomalies data presented by the National Climate Data Center (NCDC) data for the years 1880-2013. This is discussed in Refs. [28-33] and is no doubt of great societal concern in the 21st century.

The alternative viewpoint of the movement along parallels revealed by appealing to the generalized idea of a work function thus merits attention of all climate scientists. It actually means that the global average temperatures are lower now than they were in the 20th century and this is NOT a recent hiatus. The changing value (it is becoming more negative) of the nonzero intercept A, as a function of time t, in the law T = A + Bt, means that the global average temperature T is actually lower now than it would have been if temperatures had continued to rise along the parallels established in the late 19th or early 20th or the mid-20th century; see Figure 2 of [33] for the five parallels T-t graph, each parallel computed from actual (x, y) pairs, following the Millikan method. More generally, the application of Millikan method, and the generalized idea of a work function, to other fields of data analysis cannot be overlooked.

Summary and Conclusions

1. A linear law, with a nonzero intercept c, of the type $y = hx + c = h(x - x_0)$, is often observed when we analyze our (x, y) observations on a number of complex systems, such as the climate system data considered here. The nonzero c, or the cut-off $x_0 = -c/h$, in such a law is just like the nonzero work function W, conceived by Einstein, in his 1905 paper on photoelectricity. Einstein's photoelectric law was thus able to explain the cut-off frequency observed experimentally by Lenard.

2. The photoelectric law implies a movement of the empirical observations on the photoelectric effect along a family of parallel lines. A similar movement of the data along parallels is observed if we analyze our (x, y) observations carefully. The method of deducing the existence of such parallels is similar to that used by Millikan: First choose two (x, y) pairs and determine the slope. Second, prove that very nearly the same slope is observed with at least two or more (x, y) pairs in the data to conclude the existence of parallels.

3. When (x, y) observations reveal both a positive and a negative slope, the existence of a maximum point on the x-y graph can be inferred. This can be modeled by the power-exponential law, a generalization of Wien's law from radiation physics, which became the starting point of Einstein's photoelectric law and was also the basis for Planck's enunciation of quantum hypothesis.

4. A generalization of Planck-Einstein ideas is possible by attaching new meaning to the mathematical symbols U (energy) and entropy (S) in Planck's original theory. Planck is essentially describing a method of finding the average U for a complex system. This interpretation of Planck's quantum hypothesis will have wide applications outside physics, as illustrated by the present discussion of a generalization of the work function.

APPENDIX: THE PRECIPITATION DATA: SWEDEN, FINLAND, AND DENMARK

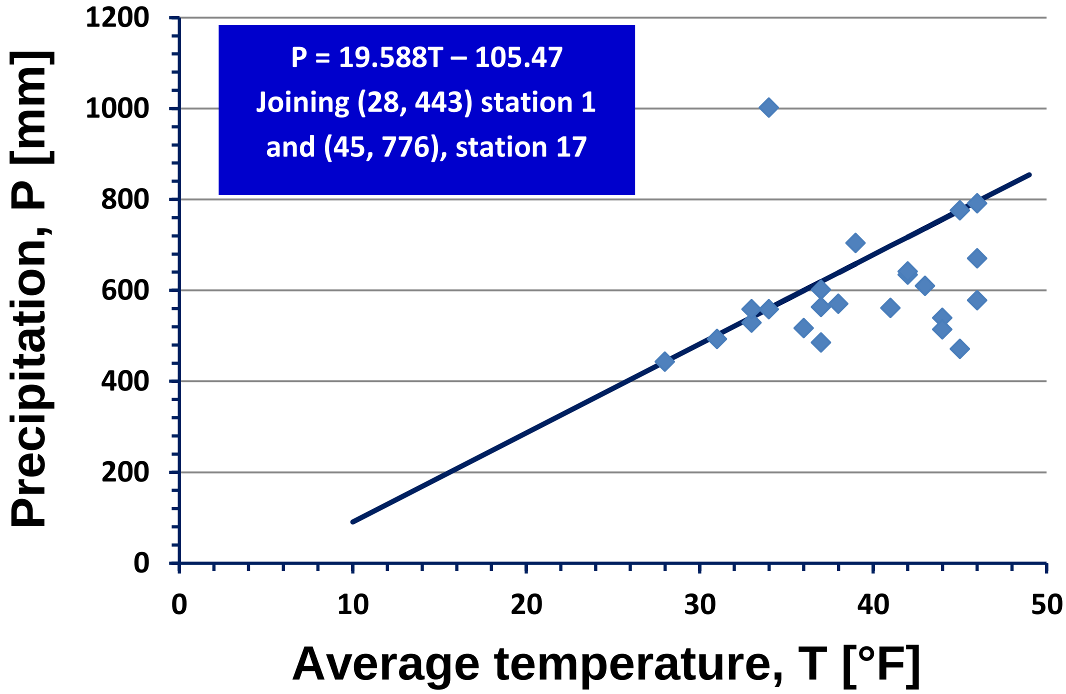

*Figure 4: Precipitation data for **23 cities of Sweden**. Ignoring the anomalous data point (34, 1002), a positive slope is revealed for the maximum precipitation, P = aT + b.*

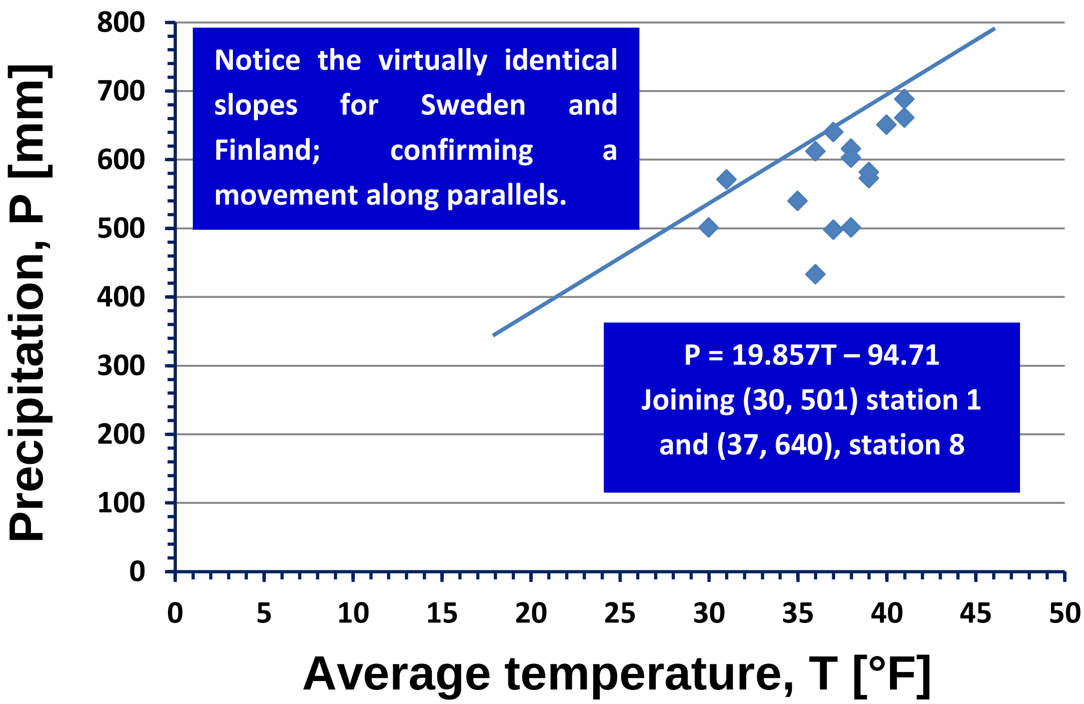

*Figure 5: Precipitation data for **15 cities of Finland**, positive slope for maximum P.*

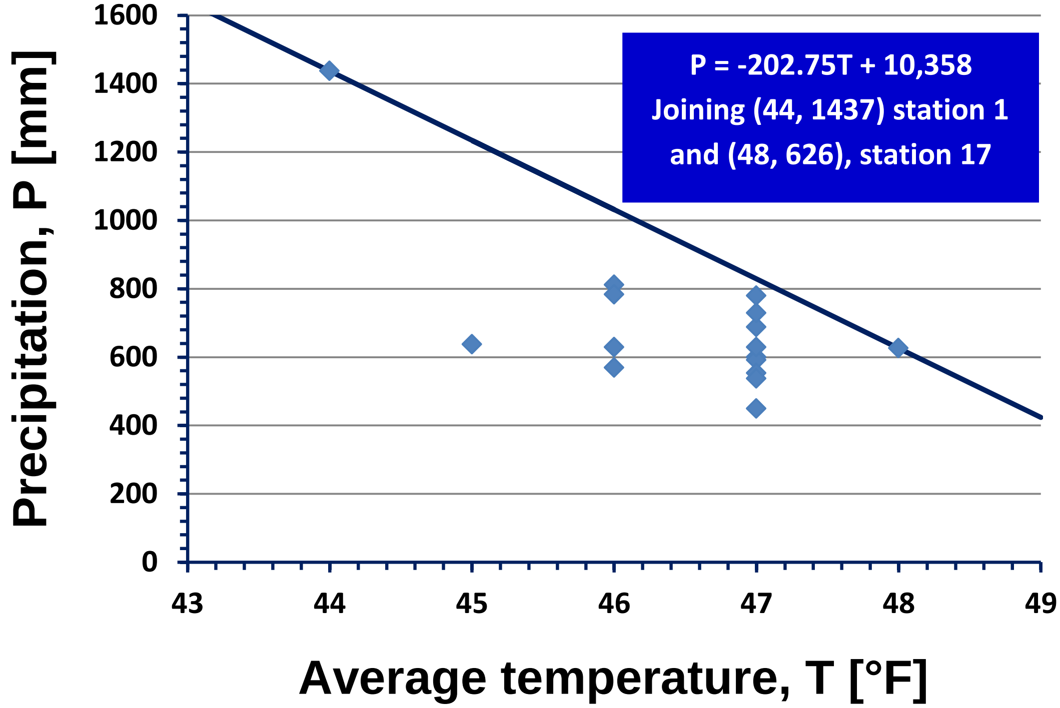

*Figure 6: Precipitation data for **17 cities of Denmark**, positive slope for maximum P. A negative slope is revealed for the **maximum** precipitation, if we include the anomalously high precipitation at the lowest temperature. The existence of both a positive and a negative slope for the P-T data means that there must be a maximum point on the P-T graph. This is obvious if we examine the composite plot in Figure 3.*

The generalized power-exponential law (with Wien's law of radiation physics being a special example), $y = mx^n e^{-ax}$ can be used to fit the data to reveal the maximum point in Figure 3. This is illustrated in Figure 7 with values of m, n, a, and a nonzero intercept c, being fitted to yield the maximum point at 33°F as revealed by the low precipitation data. There is no theoretical justification for the numerical values of these constants, other than curve fitting and some guidance from other physical problems, in particular, for the exponent "n". In Planck's law, n = 3 (with x being frequency), for Kepler's third law of planetary orbits, n = 1.5 and for Galileo's law for falling bodies with n = 2; see Longair's treatise.

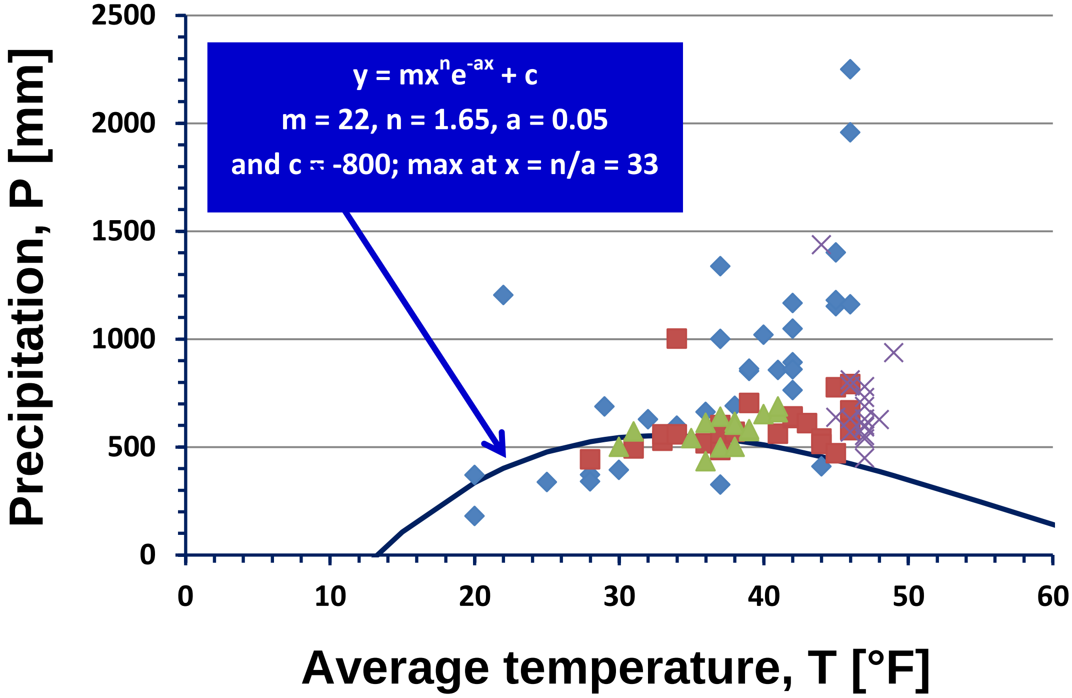

Figure 7: The power-exponential law is included here to reveal a maximum point at low precipitation levels, as the temperature increases. This combines the observations of all four Scandinavian countries. A theoretical justification, following methods used by Planck to develop quantum physics is suggested by attaching new meanings to the symbols U (energy) and S (entropy) in Planck's derivation of his law. Also, in Planck's law T = dU/dS or 1/T = dS/dU; see Shamos and Gearhart.

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  a. For lithium $V_0 = 0.4126 f - 3.593$, from 1st paper published in 1916
  b. For lithium $V_0 = 0.4223 f - 3.922$, from 2nd paper published in 1916
  c. For sodium $V_0 = 0.4069 f - 4.288$, from 2nd paper published in 1916

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